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On algebraic families of positive divisors and their associated Varieties on a projective Variety.[1)]

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In spite of their importance, little is known on algebraic families of positive cycles on a projective Variety, except for the fundamental results of Chow-v. d. Waerden on associated-forms of positive cycles.[2)] In the case of a Curve, a maximal algebraic family of divisors of a given degree form a *complete* family. But in the case of higher dimensions than 1, the situation is slightly different. A maximal algebraic family of positive divisors of a given degree on a non-singular surface is not determined uniquely in general, and there is a finite number of maximal algebraic families of the given degree, the divisors of which are mutually algebraically equivalent. A non-special linear system of a Curve belongs to the complete algebraic family such that every divisor of the family determines the complete linear system of the same dimension, which is totally contained in the algebraic family. Now the question is, *whether there exists always such a complete algebraic family on algebraic Varieties of higher dimensions*, or more precisely, *how one can obtain such a complete algebraic family from the given maximal algebraic family.*[2')] We shall show that such a family can be obtained always, by adding to the given algebraic family sufficiently large multiples of the hyperplane sections, when the ambient Variety is non-singular (th. 2). Moreover, as we shall show, algebraic families thus obtained generates the Picard Variety of the given Variety, i. e., when

1) We shall use the terminologies and conventions in A. Weil's "Foundations of Algebraic Geometry", Amer. Math. Soc. Colloq., vol. 29, 1946 and "Variétés Abéliennes et Courbes Algébrique", Act. Sc. et Ind., no. 1046.

Numbers and letters in brackets refer to Bibliography at the end of this paper.

2) Cf. [C-W] and [C-2].

2') Cf. G. Albanese, "Intorno ad alcuni concetti e teoremi fondamentali sui sistemi algebrici di curve d'una superficie algebrica." Ann. Mat. pura appl. III. s. vol. 24, 1915,