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On Killing vector fields in a Kaehlerian space.

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§0. Introduction.

S. Bochner [1, 2][1] has shown a remarkable contrast between harmonic vectors and Killing vectors in a real compact Riemannian space by proving the following theorems:

THEOREM I. *In a compact Riemannian space, there exists no harmonic (Killing) vector field, other than zero vector, which satisfies the relation*

$$R_{jk}\xi^j\xi^k \geqq 0, \qquad (R_{jk}\xi^j\xi^k \leqq 0)$$

unless we have $\xi_{j;k}=0$. *If the space has positive (negative) Ricci curvature throughout, then the exceptional case cannot arise.*

THEOREM II. *If, in a compact Riemannian space, there exist a harmonic vector field* ξ_i *and a Killing vector field* η^i, *then we have*

$$\xi_i\eta^i = \text{constant}.$$

S. Bochner has shown also a remarkable contrast between covariant analytic vectors and contravariant analytic vectors in a compact Kaehlerian space by proving the following theorems:

THEOREM III. *In a compact Kaehlerian space, there exists no self-adjoint covariant (contravariant) vector field, other than zero vector, the components of which are analytic functions of coordinates and which satisfies the relation*

$$R_{\alpha\bar{\beta}}\xi^\alpha\xi^{\bar{\beta}} \geqq 0, \qquad (R_{\alpha\bar{\beta}}\xi^\alpha\xi^{\bar{\beta}} \leqq 0)$$

unless the vector field has vanishing covariant derivative. If $R_{\alpha\bar{\beta}}\xi^\alpha\xi^{\bar{\beta}}$ *is positive (negative) definite throughout, then the exceptional case cannot arise.*

1) See the Bibliography at the end of the paper.