Journal of the Mathematical Society of Japan Vol. 4, Nos. 3～4, December, 1952.

On the perturbation theory of closed linear operators.

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(Received September 1, 1952)

The perturbation theory of linear operators has been developed by several authors. The most complete results heretofore obtained by Rellich and others[1] are mainly concerned with the "regular" perturbation of self-adjoint operators of a Hilbert space, while some attempts[2] have also been made towards the treatment of "non-regular" cases which are no less important in applications.

Recently another generalization of the theory was given by Sz.-Nagy[3]. By his elegant and powerful method of contour integration, he has been able to transfer most of the theorems for self-adjoint operators to a wider class of closed linear operators of a general Banach space.

In the meantime the present writer was studying the same problem independently and published his main results in Japanese language[4]. It now turned out[5] that there are considerable coincidences between the results as well as methods of Sz.-Nagy and those of the writer.

The purpose of the present paper is to give a further development of the theory based on the fundamental results of Sz.-Nagy and the writer. An important part will also be played in § 2 by a generalization of a method which the writer[6] used in the proof of the adiabatic theorem of quantum mechanics.

It will be pointed out that the perturbation theory of general closed linear operators is not only a generalization of that of self-adjoint operators, but the full significance of the latter is realized only in the light of the former. This is due to the fact that, whereas the function-theoretical behaviour of the eigenvalues and eigenvectors is completely revealed only when we consider the parameter ε as a complex variable, an operator $T(\varepsilon)$ regular in ε cannot in general be self-adjoint or even normal for all values of ε of a complex domain.