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Myrberg's approximation theorem on Fuchsian groups.

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Let G be a Fuchsian group of linear transformations, which make $|z|<1$ invariant and D_0 be its fundamental domain. We assume that D_0 has a finite number of sides, such that D_0 lies entirely in $|z|<1$, or has a finite number of parabolic vertices on $|z|=1$. It can be proved that this is equivalent to that the non-euclidean area of D_0 is finite[1]. Then Myrberg[2] proved the following approximation theorem.

THEOREM. *There exists a set E of measure 2π on $|z|=1$, which satisfies the following condition. Let $L=L(\theta)$ be a diameter of $|z|=1$ through $e^{i\theta}$ and L_ν $(\nu=0,1,2,\cdots)$ be its equivalents by G. Let C be any orthogonal circle to $|z|=1$. If $e^{i\theta}\in E$, then we can find ν_k, such that $L_{\nu_k}\to C\ (k\to\infty)$.*

We shall prove this theorem simply by means of Hopf's ergodic theorem.

PROOF. We denote an orthogonal circle to $|z|=1$, whose end points on $|z|=1$ are $e^{i\theta}$, $e^{i\varphi}$ by $C(\theta,\varphi)$. Now (θ,φ) can be considered as a point on a torus Ω: $0\leqq\theta\leqq 2\pi$, $0\leqq\varphi\leqq 2\pi$ and the measure mE of a measurable set E on Ω is defined by

$$mE=\iint_E d\theta\, d\varphi, \tag{1}$$

so that $m\Omega=4\pi^2$.

1) C. L. Siegel: Some remarks on discontinuous groups. Ann. of Math. 46 (1945). M. Tsuji: Theory of Fuchsian groups. Jap. Journ. Math. 20 (1952).

2) P. J. Myrberg: Ein Approximationssatz für die Fuchsschen Gruppen. Acta Math. 57 (1931).