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Conformal rigidity of Riemann surfaces.

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1. The following theorem of T. Radó [3] is well-known:

Let G be a planar region bounded by n ($\geqq 2$) Jordan curves $C_1, \cdots, C_n$, and G' be a proper subregion of G bounded by n Jordan curves $C'_1, \cdots, C'_n$, such that C'_k is homotopic to C_k in $\overline{G}$ ($k=1, \cdots, n$). Then, G admits no one-to-one conformal mapping onto G'.

In the present paper we shall consider, instead of a planar region of finite connectivity, a Riemann surface G (of finite or infinite connectivity and genus) bounded partly by a finite number of Jordan curves. Under the assumption that G admits a one-to-one conformal mapping onto a proper subregion of itself satisfying some topological conditions, we shall prove that G must be of some particularly simple structure (Theorem 2), a result which constitutes a generalization of Radó's theorem.

First, in § 2, we prove a general selection theorem on single-valued (not necessarily one-to-one) analytic mappings of a Riemann surface into another Riemann surface. In § 3, the above mentioned Theorem 2 is stated and proved, to which a remark is added in § 4. Finally, in § 5, we prove a rigidity theorem without any topological restrictions on the subregion.

2. THEOREM 1. *Let F, F^* be two Riemann surfaces whose universal covering surfaces are of hyperbolic type*[1]*, and $\{f_\nu\}_{\nu=1}^{\infty}$ be a sequence of single-valued analytic mappings of F into F^*. Then, either*

i) *there exists a subsequence $\{f_{\nu_k}\}_{k=1}^{\infty}$ which converges, uniformly in the wider sense in F (with respect to the uniform topology of F^* defined by means of Poincaré's hyperbolic metric), to a limit analytic mapping f of F into F^*; or else*

ii) *for any point p on F the point sequence $\{f_\nu(p)\}$ on F^* tends to the ideal boundary of F^* uniformly in the wider sense in F.*

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1) In point of fact, it suffices merely to assume that the universal covering surface of F^* is of hyperbolic type.