Journal of the Mathematical Society of Japan Vol. 4, Nos. 3~4, December, 1952.

On the nilpotency of nil-algebras.

By Masayoshi NAGATA

(Received June 20, 1952)

Introduction. We say in the present paper that a ring R satisfies the condition $c(n)$, if $u^n=0$ holds for all $u \in R$, n being a (given) natural number. The purpose of the present paper is to answer a problem on the nilpotency of algebras satisfying $c(n)$, raised by Y. Kawada and N. Iwahori, in proving the following

THEOREM. *Let* $\mathfrak{A}$ *be a ring with a coefficient field* K. *If* $\mathfrak{A}$ *satisfies the condition* $c(n)$ *and if* K *is of characteristic* 0, *then there exists a natural number* $f(n)$ *depending solely on* n *such that* $\mathfrak{A}^{f(n)}=0$.

In the last paragraph, we shall add some remarks concerning the case when K is a general coefficient ring.

1. Preliminaries on group rings of symmetric groups.

We denote by S_t the symmetric group on letters $1, 2, \cdots, t$; t being a natural number. Let K be the field in our theorem and $\mathfrak{o}_t$ the group ring of S_t over K.

We denote by (α) or (β) a Young diagram (of letters $1, \cdots, t$). Furthermore, for an arbitrary Young diagram (α), we denote by A_α the totality of $(-1)^{\delta(q)}q$ $(q \in S_t)$ such that i and $q(i)$ are in the same column of (α) for each i $(1 \leq i \leq t)$ and that $\delta(q)=1$ or 0 according as the permutation q is odd or even; and by S_α the totality of p $(p \in S_t)$ such that i and $p(i)$ are in the same row of (α) for each i $(1 \leq i \leq t)$. Further we set $A_\alpha^* = \sum_{a \in A_\alpha} a$ $(\in \mathfrak{o}_t)$, $S_\alpha^* = \sum_{s \in S_\alpha} s$ $(\in \mathfrak{o}_t)$.

Remark. A_α is a subgroup of $S_t \times \{1, -1\}$ and S_α is a subgroup of S_t.

Now the following facts are well known:

(1) $\mathfrak{o}_t S_\alpha^* A_\alpha^*$ is a simple left ideal of $\mathfrak{o}_t$.

(2) Every simple left ideal of $\mathfrak{o}_t$ is operator isomorphic to $\mathfrak{o}_t S_\alpha^* A_\alpha^*$ with a suitable (α).

From these facts follows easily