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Theorems in the geometry of numbers for Fuchsian groups.

By Masatsugu TSUJI

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1. We introduce a non-euclidean metric in $|z|<1$ by

$$ds=\frac{2|dz|}{1-|z|^2},$$

so that the non-euclidean radius r of a circle $|z|=\rho<1$ is

$$r=\log\frac{1+\rho}{1-\rho} \tag{1}$$

and the non-euclidean measure $\sigma(E)$ of a measurable set E in $|z|<1$ is

$$\sigma(E)=\iint_E \frac{4\,rdrd\theta}{(1-r^2)^2} \qquad (z=re^{i\theta}),$$

hence the non-euclidean area of a disc $\Delta:|z|\leqq r<1$ is

$$\sigma(\Delta)=\frac{4\pi r^2}{1-r^2}. \tag{2}$$

Let G be a Fuchsian group of linear transformations, which make $|z|<1$ invariant and D_0 be its fundamental domain. Let E be a measurable set in $|z|<1$ and E_ν $(\nu=0,1,2,\cdots)$ be its equivalents by G and $A(r,E_\nu)$ be the non-euclidean measure of the part of E_ν contained in $|z|\leqq r$ and put

$$A(r,E)=\sum_{\nu=0}^{\infty}A(r,E_\nu). \tag{3}$$

If $\sigma(D_0)<\infty$, then I have proved in another paper[1)] that

$$\int_0^r \frac{A(r,E)}{r}\,dr=\frac{2\pi\,\sigma(E)}{\sigma(D_0)}\log\frac{1}{1-r}+O(1) \qquad (r\to 1). \tag{4}$$

1) M. Tsuji: Theory of Fuchsian groups. Jap. Journ. Math. 21 (1951).