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A generalization of Cartan space.

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A space such that the area of a domain on a hypersurface $x^i=x^i(u^\alpha)$, $\alpha=1,2,\cdots,n-1$, is given by the $(n-1)$-ple integral

$$\int_{(n-1)} F(x^i, \partial x^i/\partial u^\alpha)\, du^1 \cdots du^{n-1}$$

is called a Cartan space.. E. Cartan [1] has shown that this space may be regarded as a manifold of the hyperplane elements $(x^i, \partial x^i/\partial u^\alpha)$. Thereafter L. Berwald has treated the geometry at large of this space, and T. Okubo and the present auther have extended this geometry to the higher order $(n-1)$-ple integrals of some special forms. In this paper the auther will establish the geometry of a space in which the area of a domain on a K-dimensional surface $x^i=x^i(u^\alpha)$, $i=1,2,\cdots,n$; $\alpha=1,\cdots,K$, is given by the K-ple integral

$$\int_K F(u^\alpha, x^i, \partial x^i/\partial u^\alpha, \partial^2 x^i/\partial u^\alpha \partial u^\beta)\, du^1 \cdots du^K .$$

It is convenient to regard the space in question as a manifold of the K-dimensional surface elements of the third order and u^α, $(\alpha=1,\cdots, K)$ which we shall denote by $F_n^{(3)}$. Namely, the manifold $F_n^{(3)}$ consists of all system of values of $u^\alpha, x^i, \partial x^i/\partial u^\alpha, \partial^2 x^i/\partial u^\alpha \partial u^\beta, \partial^3 x^i/\partial u^\alpha \partial u^\beta \partial u^\gamma$.

Throughout this paper we shall use the notations

$$X^i_a=\frac{\partial x^i}{\partial \bar{x}^a}, \quad X^a_i=\frac{\partial \bar{x}^a}{\partial x^i}, \quad X^i_{a(2)}=\frac{\partial^2 x^i}{\partial \bar{x}^{a_1}\partial \bar{x}^{a_2}}, \quad X^a_{i(2)}=\frac{\partial^2 \bar{x}^a}{\partial x^{i_1}\partial x^{i_2}}, \cdots,$$

$$U^\alpha_\lambda=\frac{\partial u^\alpha}{\partial \bar{u}^\lambda}, \quad U^\lambda_\alpha=\frac{\partial \bar{u}^\lambda}{\partial u^\alpha}, \quad U^\alpha_{\lambda(2)}=\frac{\partial^2 u^\alpha}{\partial \bar{u}^{\lambda_1}\partial \bar{u}^{\lambda_2}}, \quad U^\lambda_{\alpha(2)}=\frac{\partial^2 \bar{u}^\lambda}{\partial u^{\alpha_1}\partial u^{\alpha_2}}, \cdots,$$

$$U^{\alpha(s)}_{\lambda(s)}=U^{\alpha_1}_{\lambda_1} U^{\alpha_2}_{\lambda_2} \cdots U^{\alpha_s}_{\lambda_s}, \qquad U^{\lambda(s)}_{\alpha(s)}=U^{\lambda_1}_{\alpha_1} U^{\lambda_2}_{\alpha_2} \cdots U^{\lambda_s}_{\alpha_s}.$$

which are evaluated for the transformations