

On the automorphisms of a real semi-simple Lie algebra.

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Let \mathfrak{G} be a real semi-simple Lie algebra and G the group of automorphisms of \mathfrak{G} . As is well-known, the connected component G^0 of the identity of G coincides with the group of inner automorphisms of \mathfrak{G} . The object of this paper is to investigate the structure of the factor group G/G^0 ; i. e. the type of outer automorphisms of \mathfrak{G} modulo the inner automorphisms.

E. Cartan [2]¹⁾ has treated our subject in 1927, and established intimate relations between the group G and the group of isometric transformations in a certain type of Riemannian space. However, the problem being algebraic in its character, a more direct treatment might be desirable. Now, Cartan [1] had earlier dealt with the same subject by a more algebraic method for the case where the Lie algebra \mathfrak{G} is complex semi-simple. F. Gantmacher [4] [5] attempted to rearrange Cartan's results concerning complex semi-simple Lie algebras on the basis of the structure theory founded by H. Weyl [9], and then applied so obtained results to the classification of real simple Lie algebras which had been also achieved by Cartan with specific devices. It is to be noted that recently I. Satake [8] has given an algebraic proof to a theorem of Cartan on which Gantmacher had yet to depend.²⁾ In following this algebraic direction, we shall give in this paper some results analogous to the case of complex semi-simple Lie algebras.

In §1, we reduce our problem to that concerning a maximal compact subgroup of the group G . This reduction was done by Cartan [3], but we shall perform it by means of a method suggested by G. D. Mostow [7]. Next, to investigate the structure of this maximal compact

1) Numbers in brackets refer to Bibliography at the end of this paper.

2) See Footnote 9).