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# An application of Ahlfors's theory of covering surfaces.

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We shall give here an alternative proof of the following theorem of Ahlfors[1)] using his theory of covering surfaces.[2)]

THEOREM. *Let* $w=f(z)$ *be a meromorphic function in* $|z|<R$, *and* $D_1, D_2, \cdots, D_q$ $(q \geqq 3)$ *be simply connected closed domains on the Riemann sphere lying outside each others. If*

$$R \geqq k\,\frac{1+|f(0)|^2}{|f'(0)|},$$

*k being a constant depending only on* $D_i$ $(i=1,2,\cdots,q)$, *then we have*

$$\sum_{i=1}^{q}\left(1-\frac{1}{\mu_i}\right) \leqq 2,$$

$f(z)$ *ramifying at least* $\mu_i$*-ply on* $D_i$ $(i=1,2,\cdots,q)$.[3)]

PROOF. Suppose that the latter inequality does not hold. Then, since $\mu_i$ are positive integers or $=\infty$, it is easily verified that there holds for any $r\,(\leqq R)$

$$\sum_{i=1}^{q}\left(1-\frac{1}{\mu_i(r)}\right) \geqq \sum_{i=1}^{q}\left(1-\frac{1}{\mu_i}\right) \geqq 2+\frac{1}{42}, \tag{1}$$

where $f(z)$ ramifies at least $\mu_i(r)$-ply on $D_i$ $(i=1,2,\cdots,q)$ in $|z| \leqq r \leqq R$ $\left(\mu_i(r) \geqq \mu_i(R)=\mu_i\right)$.

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1) L. Ahlfors, Sur les domaines dans lesquels une fonction méromorphe prend des valeurs appartenant à une région donnée. (Acta Soc. Sci. Fenn. N. s. 2 Nr. 2 (1933)).

2) L. Ahlfors, Zur Theorie der Überlagerungsflächen (Acta Math. 65 (1935)); or R. Nevanlinna, Eindeutige analytische Funktionen.

3) By this expression we mean that the Riemann image of $|z|<R$ by $f(z)$ contains no connected island above $D_i$ whose number of sheets is $<\mu_i$.