Journal of the Mathematical Society of Japan Vol. 4, No. 1, July, 1952.

On the remainder term of Nevanlinna's second fundamental theorem.

By Masatsugu TSUJI

(Received March 27, 1951)

1. Let $w(z)$ be meromorphic in $|z|<R$ $(\leqq\infty)$ and $n(r, a)$ be the number of zero points of $w(z)-a$ in $|z|<r$ and

$$[a, b]=\frac{|a-b|}{\sqrt{(1+|a|^2)(1+|b|^2)}}.$$

We put

$$m(r, a)=\frac{1}{2\pi}\int_0^{2\pi}\log\frac{1}{[w(re^{i\theta}), a]}\,d\theta,$$

$$N(r, a)=\int_0^r\frac{n(r, a)-n(0, a)}{r}\,dr+n(0, a)\log r+C,$$

where C is a constant, which is determined by $\lim_{r\to 0}\big(m(r, a)+N(r, a)\big)=0$.

Then $T(r)=m(r, a)+N(r, a)$ is independent of a, which is the Nevanlinna's first fundamental theorem.

Let $n_1(r, 0)$, $n_1(r, \infty)$ be the number of zero points and poles of $w'(z)$ in $|z|<r$ and $n_1(r)=n_1(r, 0)-n_1(r, \infty)+2n(r, \infty)\geqq 0$,

$$N_1(r)=\int_0^r\frac{n_1(r)-n_1(0)}{r}\,dr+n_1(0)\log r,$$

Then

NEVANLINNA'S SECOND FUNDAMENTAL THEOREM:

$$(q-2)\,T(r)\leqq\sum_{i=1}^{q}N(r, a_i)-N_1(r)+\Omega(r)\qquad(q\geqq 3),$$

where the remainder term $\Omega(r)$ *satisfies the following condition:*

(i) *If* $R=\infty$,

$$\Omega(r)=O\big(\log r+\log T(r)\big),\tag{I}$$