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A remark on the prolongation of Riemann surfaces of finite genus.

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Let F be an abstract Riemann surface. If there exists no one-valued, regular analytic and non-constant function on F such that its Dirichlet integral taken over F is finite, we shall say that F is a *surface of class* $N_{\mathfrak{D}}$ (F has "einen hebbaren Rand" in Sario's terminology[1]).

If F is of finite genus p, we can map F conformally onto a part $\bar{F}$ of a closed Riemann surface F^* of the same genus[2]. Then, Nevanlinna stated the following conjecture[3]:

THEOREM. *The prolongation of a Riemann surface F of finite genus p onto a closed Riemann surface F^* is unique, if and only if F is a surface of class* $N_{\mathfrak{D}}$.

The "uniqueness" means: if F is mapped conformally onto a part $\bar{F}$ of F^* and a part $\bar{F}_1$ of F_1^* respectively, then the analytic function which maps $\bar{F}$ onto $\bar{F}_1$ maps necessarily F^* onto F_1^*.

This conjecture was proved by Ahlfors and Beurling[4] for the case $p=0$: *A plane region Ω is of class* $N_{\mathfrak{D}}$ *if and only if every univalent (schlicht) function in Ω is linear.* In this note we shall show that the conjecture for an arbitrary p can be easily proved by means of this Ahlfors-Beurling's theorem.

Let E be a bounded closed set of points on the complex z-plane. If any one-valued regular analytic function in a neighbourhood $U-E$ of E with finite Dirichlet integral taken over $U-E$ is regular also on E, we shall say, for convenience' sake, that E is a *null-set of class* $N_{\mathfrak{D}}$[5].

We cut F along a non-decomposing system of p analytic loop cuts on F having no points in common with each others, and map the resulting surface of planar character (schlichtartig) conformally onto a domain D on the z-plane, which is bounded by $2p$ closed analytic curves C_i, C_i' ($i=1, \cdots, p$) and a bounded closed set of points E, so