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Theory of the Spherically Symmetric Space-Times, I
Characteristic System

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§ 1. Definition

A spherically symmetric space-time is a 4-dimensional Riemannian space whose fundamental form is reducible to

$$ds^2 = -A(r,\ t)dr^2 - B(r,\ t)(d\theta^2 + \sin^2\theta d\phi^2) + C(r,\ t)dt^2 \qquad (1\cdot1)$$

where A, B and C are any positive valued functions of r and t. Historically $(1\cdot1)$ was obtained by generalizing the metric of the Minkowski space-time. Eiesland defined this space-time from the standpoint of the group of motions using the group of ordinary 3-dimensional rotations.[(1)] In this paper, (1) we shall give a new definition of the s. s. (spherically symmetric) space-time S_0 using some tensor equations to be satified by g_{ij}. (2) At the same time we shall define a set of vectors and scalars characterizing this space-time. (3) Then we shall show that this new definition coincides with Eiesland's one. (4) Finally we shall obtain some properties of the s. s. space-time.

Definition : *Spherically symmetric space-time* is a 4-dimensional Riemannian space with the following properties :

(I) Its curvature tensor satisfies the equation

$$K_{ijlm} = -\overset{1}{\rho}\alpha_{[i}\alpha_{[l}\beta_{j]}\beta_{m]} - \overset{2}{\rho}g_{[i[l}\alpha_{j]}\alpha_{m]} + \overset{3}{\rho}g_{[i[l}\beta_{j]}\beta_{m]} + \overset{4}{\rho}g_{[i[l}g_{j]m]} \qquad (F_1)$$

where α_i and β_i are mutually orthogonal unit vectors (real or complex) satisfying

$$\nabla_i\alpha_j = \sigma\alpha_i\beta_j + \varkappa(g_{ij} + \alpha_i\alpha_j - \beta_i\beta_j) + \bar{\sigma}\beta_i\beta_j \qquad (F_2)$$

$$\nabla_i\beta_j = \bar{\sigma}\beta_i\alpha_j + \bar{\varkappa}(\qquad ,, \qquad) + \sigma\alpha_i\alpha_j \qquad (F_3)$$

$$\alpha_s\alpha^s = -1,\ \beta_s\beta^s = 1,\ \alpha_s\beta^s = 0 \qquad (1\cdot2)$$