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A Generalization of Laguerre Geometry, II.

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§ 8. Group of holonomy.

We can define the group of holonomy in our space, by developing a tangent hypersphere space along a closed curve as in the case of Riemannian space. Let us consider the relations between the group of holonomy and the structure of our space. In the first place we study the case in which the group of holonomy fixes a hypersphere. Next we consider the case in which the group of holonomy fixes two independent hyperspheres.

(A) The case in which the group of holonomy fixes the hypersphere of the form

$$V^{\lambda}=\begin{cases}V^{i} & (\lambda=i)\\ 0 & (\lambda=0).\end{cases}$$

In this case we have

$$(8\cdot 1)\qquad \delta V^{\lambda}+dx^{\lambda}=dV^{\lambda}+\Gamma^{\lambda}_{\mu k}V^{\mu}dx^{k}+dx^{\lambda}=0.$$

That is

$$(8\cdot 2)\qquad \partial V^{i}/\partial x^{k}+\Gamma^{i}_{jk}V^{j}+\delta^{i}_{k}=0,\quad \Gamma^{0}_{jk}V^{j}=0.$$

Hence the vector V^{i} forms so-called* *concurrent vector field.*

(B) The case in which the group of holonomy fixes the hypersphere of the form

$$V^{\lambda}=\begin{cases}0 & (\lambda=i)\\ V^{0} & (\lambda=0).\end{cases}$$

In this case we have

$$(8\cdot 3)\qquad \delta V^{\lambda}+dx^{\lambda}=\begin{cases}\Gamma^{i}_{ok}V^{0}dx^{k}+dx^{i}=0, & (\lambda=i)\\ dV^{0}=0 & (\lambda=0).\end{cases}$$

Since dx^{i} are arbitrary, we get

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* K. Yano: Sur le parallélisme et la concourance dans l'espace de Riemann. Proc. Imp. Acad. Tokyo, 19 (1943), pp. 189–197.