Journal of the Mathematical Society of Japan Vol. 3, No. 1, May, 1951.

Some Remarks concerning p-adic Number Field

Tadao TANNAKA

The content of this paper is already published in 1942 in Japanese ([1]) but as after that time several remarks and developments in this direction were made by other authors, most of them being also Japanese, it would not be meaningless to translate the results in European language. Especially Nakayama, who obtained a generalizaton of my result, hoped me to quote his result together, if the translation is attempted in future.

1. The results obtained by myself are the following two theorems. In this paper all the concerning fields are p-adic number fields and $G(k/K)$ denote the group of numbers A in K such that $N_{Kk}(A)=1$.

Theorem 1. (*Ordering theorem*). *If the fields K/k and K/k' are both relative abelian, then the necessary an a sufficient condition that $k'\subset k$ holds is $G(k/K)\subset G(k'/K)$.*

Theorem 2. *If K/k is abelian of degree n and a factor set of exponent n, then the group $G(k/K)$ is generated by the numbers of the form $a_{\sigma,\tau}/a_{\tau,\sigma}$ and $b^{1-\sigma}$:*

$$G(k/K)=\{a_{\sigma,\tau}/a_{\tau,\sigma},\ K^{1-\sigma}\}.$$

I called the latter theorem "Hauptgeschlechtssatz im Minimalen". Clearly this is a generalization of the well-known Hilbert's norm theorem concerning cyclic fields.

2. Next I will give a brief sketch of the further developments obtained by other authors.

T. Nakayama [2] generalized the theorem 2 as follows.

Theorem 2'. *If K/k is a normal field with the Galois group $\mathfrak{G}=\{\rho,\sigma,\tau,\dots\}$, then the group $G(k/K)$ consists of the products of the elements*

$$b^{1-\rho}(b\in K,\ \rho\in\mathfrak{G})$$

and

$$b(\sigma,\tau)/b(\tau,\sigma)\quad (\textit{where } (b)\sim(a) \textit{ and } b(\sigma\tau,\mathfrak{G})=b(\tau\sigma,\mathfrak{G})).$$

Thereby (a) is a fixed factor set corresponding to a division algebra.