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Algebraic Correspondences between Algebraic Varieties

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Let V^n be a complete Variety without multiple Point in the algebraic geometry with the universal domain of all complex numbers. Let k be the smallest field of definition for V, then to every V-divisor Y is attached the smallest extension $k(P)$ of k over which Y is rational. In view of Zariski's results it would not be too restrictive to assume that P is a generic Point of a suitable complete Variety U^m without multiple Point, which is defined over the algebraic closure K of k in $k(P)$. There exists then a $(U\times V)$-divisor X, which is rational over K, such that

$$X\cdot(P\times V) = P\times Y.$$

We call such a X a *correspondence between* U *and* V, since it is a correspondence in the classical sense if both U and V are curves. In the above connection a problem concerning V-divisors can be translated into a problem on correspondences and vice versa. The V-divisor Y varies in a linearly equivalent system for the variable Point P when and only when X is of the form

$$X = Y_1\times V + U\times Y_2 + (\varphi),$$

where Y_1 is a U-divisor, Y_2 a V-divisor and φ a function on $U\times V$. We call such a X a *correspondence with valence zero* in agreement with the case of curves. Since such correspondences form a submodule in the module of all correspondences, we can consider their residue-class module. We call this module the *module of correspondences* and we denote it by $C(U, V)$.

In this paper we shall assume that both U and V have non-singular projective models, which, we hope, may not be a restriction. Let then

$$\Phi_{\alpha i}\ (1 \leqq i \leqq q_\alpha)$$

be a base of the Picard differentials of the first kind, and let

$$\gamma_{\alpha i}\ (1 \leqq i \leqq 2q_\alpha)$$

We shall use freely the results and terminologies in Weil's book: Foundations of algebraic geometry, Am. Math. Soc. Colloq., Vol. 29 (1946). About the present paper the author has received kind remarks from Prof. Weil to whom he express his best thanks.