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# On a System of Differential Equations

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## 1. Establishment of Problem.

Notation: A great roman letter means a matrix of the type $(m, n)$.
Problem: Let $D$ be a domain with the boundary $\Gamma$ in the space $R$ of variable point $(x_1, x_2, \cdots)$. We want to find a real continuous function-matrix $U$ satisfying the following conditions;

$$\Delta U + KU = 0 \quad \text{in } D$$

$$\frac{dU}{dn} + UH = 0 \quad \text{on } \Gamma,$$

where $\Delta = \sum_i \frac{\partial^2}{\partial x_i^2}$ and $\frac{d}{dn}$ normal derivation, each applying to every element of $U$, $K^{(m)}$ is a constant symmetric matrix and $H^{(n)}$ is a constant positive definite symmetric matrix.*)

Such a function-matrix is called a harmonic function-matrix in $D$.

By two matrices $U$, $V$ holds Green's formula,

$$(1) \quad \int_\Gamma U \frac{dV'}{dn} dw = \int_D U \Delta V' dv + \int_D \sum_i \frac{\partial U}{\partial x_i} \frac{\partial V'}{\partial x_i} dv,$$

$$(2) \quad \int_\Gamma \left( U \frac{dV'}{dn} - \frac{dU}{dn} V' \right) dw = \int_D (U \Delta V' - \Delta U \cdot V') dv,$$

where $'$ means transposition of a matrix.

When $U$ is harmonic, from (1) follows

$$\int_\Gamma U \frac{dU'}{dn} dw = \int_D U \Delta U' dv + \int_D \sum_i \frac{\partial U}{\partial x_i} \frac{\partial U'}{\partial x_i} dv,$$

$$\left( \int_D U U' dv \right) K = \int_\Gamma U H U' dw + \int_D \sum_i \frac{\partial U}{\partial x_i} \frac{\partial U'}{\partial x_i} dv.$$

Now put for arbitrary two matrices $U$, $V$,

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*) As the content of the present problem is of rather formal interest, we may make, suitable assumptions about domains, existence of derivatives and continuity as it needs.