Journal of the Mathematical Society of Japan Vol. 3, No. 1, May, 1951.

Theorems of Bertini on Linear Systems

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As the fundamental theorems of the classical algebraic geometry we have these of Bertini:

I. *The general section U_{r-1} of an algebraic variety U_r by a linear system without fixed components is irreducible, provided that the linear system is not composed of an algebraic pencil.*

II. *The general section U_{r-1} of U_r by a linear system can not have any singular points outside the singlar points of U_r and outside the base points of the linear system.*

The first proposition was proved purely algebraically first by Zariski,[1] when the basic field k of U_r is of characteristic $p=0$. Matsusaka[2] remarked that this holds even when $p>0$ under an additional condition.

Zariski[3] has also given an adequate formulation to the second proposition for the case $p>0$, as it cannot be maintained in the above formulation in this case.

In this paper we shall study how the above formulation will not be maintained when $p>0$, and will give a sufficient condition that it should be maintained. Thereby we shall give also a new proof the first proposition. Further we shall add a new elementary proof of the second proposition in the classical case.[4]

1. Let U_r be an r-dimensional irreducible algebraic variety immersed in an N-dimensional projective space S^N and defined over a field k of characteristic $p \geqq 0$. We denote by $(\xi_0, \xi_1, \dots, \xi_N)$ the homogeneous coordinates of the generic point of U/k. And we assume that the linear system on U

$$\lambda_0 f_0(\xi) + \lambda_1 f_1(\xi) + \dots + \lambda_m f_m(\xi) \tag{1}$$

has no fixed components.

1) See Zariski [1].

2) See Matsusaka [5].

3) See Zariski [2].

4) We shall use the same terminalogies in Weil's book [3].