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On the Group of Automorphisms of a Function Field

Kenkichi IWASAWA and Tsuneo TAMAGAWA

§ 1. Let K be an algebraic function field over an algebraically closed constant field k. It is well-known that the group of automorphisms of K over k is a finite group, when the genus of K is greater than 1. In the classical case, where k is the field of complex numbers, this theorem was proved by Klein and Poincaré[1] by making use of the analytic theory of Riemann surfaces. On the other hand, Weierstrass and Hurwitz gave more algebraical proofs in the same case[2], which essentially depend upon the existence of so-called Weierstrass points of K. Because of its algebraic nature, the latter method is immediately applicable to the case of an arbitrary constant field of characteristic zero. In the case of characteristic $p \neq 0$, H. L. Schmid proved the theorem along similar lines[3]; the proof being based upon F. K. Schmidt's generalization of the classical theory of Weierstrass points in such a case[4].

Now it has been remarked, since Hurwitz, that the representation of the group G of automorphisms of K by the linear transformations, induced by G in the set of all differentials of the first kind of K, is very important for the study of the structure of G. The purpose of the present paper is to show that we can indeed prove the finiteness of G by the help of such a representation instead of the theorem on Weierstrass points. In the next paragraph we analyze the structure of the subgroup $G(p)$ of G, consisting of those automorphisms of K, which leave a given prime divisor P of K fixed, where K may be any function field of genus greater than zero. The finiteness of $G(p)$ is also proved by H. L. Schmid; but his proof depends essentially upon formal calculations of polynomials, whereas our method is more group-theoretical. In the last paragraph we then prove our theorem by considering the above mentioned representation of G and by using a theorem of Burnside on irreducible groups of linear transformations.

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1) Cf. Poincaré [3]
2) Cf. Weierstrass [6] and Hurwitz [2]
3) Cf. H. L. Schmid [4]
4) Cf. F. K. Schmidt [5]