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Affine and Projective Geometries of System of Hypersurfaces

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§ 1. *Introduction.* J. Douglas [1]* studied affine and projective geometries of a space of K-spreads, K-spreads being given by a system of partial differential equations of the form

$$\frac{\partial^2 x^i}{\partial u^\beta \partial u^\gamma}+H^i_{\beta\gamma}(x\,;p)=0, \qquad \left(p^i_\alpha=\frac{\partial x^i}{\partial u^\alpha}\right), \tag{1·1}$$

$$(i, j, k, \ldots\ldots =1, 2, \ldots\ldots, N;\ \alpha, \beta, \gamma, \ldots\ldots =1, 2, \ldots\ldots, K)$$

where $H^i_{\beta\gamma}=H^i_{\gamma\beta}$ are homogeneous function system of p with respect to the lower indices β and γ.

The problem to determine a privileged projective connection with respect to which the system of K-dimensional flat subspaces coincides exactly with that of K-spreads given by (1·1) was studied by S. S. Chern [3], Chih-Ta Yen [6] and the present authors [4], [5].

In a space of K-spreads, we consider that the elements of the space are the points and the K-dimensional linear spaces at each point. The point is represented by its coordinates (x^i) and the K-dimensional linear space by K linearly independent contravariant vectors (p^i_α) contained in it. But, in a space of $(n-1)$-spreads, the hyperplane element may be represented by a covariant vector (u_i). Thus the geometry of $(n-1)$-spreads may be studied by a method some what different from the general one. The case of $(n-1)$-spreads was once treated by M. Hachtroudi [2], but we shall retake this case and study it by a method shown in [5].

Suppose that there be given, in an N-dimensional space X_N referred to a coordinate system (x^i), a system of hypersurfaces in such a way that there exists one and only one hypersurface passing through N points given in general position, and belonging to the system. Such a system of hypersurfaces is given by a finite equation of the form

$$f(x^1, x^2, \ldots\ldots, x^N; a^1, a^2, \ldots\ldots, a^N)=0, \tag{1·2}$$

* See the Bibliography at the end of the paper.