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On the Class Field Theory on Algebraic Number Fields with Infinite Degree

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By the celebrated Takagi's class field theory a finite normal abelian extension field K_0 over an algebraic number field k_0 with finite degree is completely characterized by the corresponding ideal group $H(K_0/k_0)$ mod. $\mathfrak{f}$. (Cf. Takagi [9]). Using the notion of "idèle" Chevalley has reformed the class field theory so that we can characterize the Galois group $G(\tilde{k}_0/k_0)$ of the maximal abelian extension $\tilde{k}_0$ of k_0 by a suitable factor group of the group $J(k_0)$ of all the idèles. (Cf. Chevalley [1], [2], Weil [11]).

On the other hand the ideal theory of an algebraic number field k with infinite degree was investigated by many authors (Herbrand [4], Krull [6], Moriya [7] and others). Especially Moriya [8] has extended the Takagi's class field theory on such field k. Nevertheless the idèle theory on such field k does not yet appear in the literature. The aim of this note is to extend the Chevalley's idèle theory on algebraic number fields with infinite degree and to reform the class field theory established by Moriya. Our chief method is to consider the inductive limit group of the idèle groups $J(k_\lambda)$ of algebraic number fields $k_\lambda \subset k$ with finite degree.

1. Let P be the rational number field, and k be an algebraic number field over P with infinite degree. We shall denote by $k_\lambda (\lambda \in \Lambda)$ the fields which are subfields of k and have finite degree over P. We have then $k = \cup_{\lambda \in \Lambda} k_\lambda$.

Now we shall define a semi-order

$$\lambda < \mu \qquad \text{for } k_\lambda \subset k_\mu \tag{1}$$

in Λ, then Λ becomes a directed set.

By a prime divisor $\mathfrak{p}$ of k we shall mean as usual an equivalence class of valuations of k. A valuation of k induces a valuation of $k_\lambda \subset k$, and so a prime divisor $\mathfrak{p}$ of k determines a unique prime divisor $\mathfrak{p}_\lambda$ of $k_\lambda \subset k$. We shall denote it by

$$\mathfrak{p}_\lambda = \pi_\lambda \mathfrak{p} \qquad (\lambda \in \Lambda). \tag{2}$$