Journal of the Mathematical Society of Japan Vol. 2, Nos. 3-4, March, 1951.

On a Theorem of E. Cartan.

Ichiro SATAKE.

(Received Oct. 30, 1950)

Introduction

In 1913, E. Cartan proved two remarkable theorems which give a penetrating method for the determination of all irreducible representations of semi-simple complex Lie algebras [2].[1] These theorems can be formulated as follows:

I. *There exists a one-to-one correspondence between the irreducible representations of a given semi-simple Lie algebra* $\tilde{L}$ *and the highest (possible) weights of* $\tilde{L}$.

Thus every highest possible weight ξ is actually a highest weight of some irreducible representation P of $\tilde{L}$ and conversely P is uniquely determined by ξ up to equivalence. (In this sense, we write $P=P_{\xi}$.)

II. *All the highest (possible) weights of* $\tilde{L}$ *form a semi-group, isomorphic to the direct product of* n *semi-groups, each of which is formed of all non-negative integers,* n *denoting the rank of* $\tilde{L}$, *i. e. the (complex) dimension of maximal abelian subalgebras in* $\tilde{L}$.

Thus the sum $\xi+\xi'$ of two highest possible weights ξ and ξ' of $\tilde{L}$ is again a highest possible weight of $\tilde{L}$ and the corresponding irreducible representation $P_{\xi+\xi'}$ is composed from those of ξ and ξ' through a definite process. (We call it the *Cartan composite* of P_{ξ} and $P_{\xi'}$) In this manner, all the irreducible representations of $\tilde{L}$ can be generated from n of them, called *fundamental*.

The essential part of these theorems consists in the existence of an irreducible representation for every highest possible weight (in I), and in that of n fundamental weights for the semi-group (in II). Cartan's original proof deals separately with the different types of simple algebras. So his proofs of I and II both depend on his former results on the classification of simple Lie algebras [1]. Shortly afterwards, H. Weyl [10] remarked that Theorem I might be obtained without the classification-theory by means of the completeness of prime characters of compact