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On the Differentability of the Unitary Representation of the Lie Group.

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J. von Neumann ([2]) has introduced the notion of the differentiablity of the matric group, and given a method of forming Lie algebras of matric Lie groups. This notion was extended further by K. Yosida ([4]) to the group embedded in tne normed ring.

In this paper, we shall utilize this idea to form the Lie algebra for the Lie group G embdeded in the unitary group with the weak topology in the Hilbert space $\mathfrak{H}$. Namely we shall show that the set of all operators

$$\tilde{A}_{\sigma(t)}=\lim_{t\to 0}\frac{U_{\sigma(t)}-E}{t}$$

for each one-parameter subgroup $\sigma(t)$ of G, forms the Lie algebra of G in a sense to be specified in Theorem 2, 3 below. There is diffculty on the domains of these operators. We shall show that they have a meet everywhere dense in $\mathfrak{H}$. By the way we obtain a new proof of M. H. Stone's theorem on the one-parameter group of unitary operators.

In § 1 we give a résumé of the theory of simple unitary structures, which we shall need in the proof of the fact that the domains of $\tilde{A}_{\sigma(t)}$ have an everywhere dense meet. § 2 contains a lemma that every element of $L^1(G)$ is approximable by C^2 functions. Our main results are Theorem 2, 3 in § 3.

§ 1.

Let G be a Lie group. We denote elements of G with σ, τ,$\cdots$. On the other hand let $\mathfrak{H}$ be a Hilbert space, x, y... elements of $\mathfrak{H}$. A continuous unitary representation of G is a continuous homomorphic mapping $\sigma\to U_\sigma$ into the group of all unitary operators defined on $\mathfrak{H}$ and provided with the weak topology. The pair $\{U_\sigma,\mathfrak{H}\}$ is then called a *unitary structure* of G. If $\{U_\sigma,\mathfrak{H}\}$ is a unitary structure of G and if, moreover, there is