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On the Differential Forms of the First Kind on Algebraic Varieties II.

Shoji KOIZUMI.

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We shall give some supplementary remarks to my previous paper on the same subject[1]. As in Weil's definition, we shall call differential forms to be of the first kind on a Variety $\boldsymbol{U}$, when they are finite at every simple Point on every Variety birationally equivalent to $\boldsymbol{U}$. This definition is equivalent to my previous one in $[K]$, if $\boldsymbol{U}$ has a birationally equivalent model which is a complete Variety without singularities.

1. We shall prove the following theorem as an extension of the theorem 2 in $[K]$.

THEOREM 1. Let ω be a differential form of the first kind of degree r on a Product-Variety $\boldsymbol{U}\times\boldsymbol{V}$, then we have the following expression

$$\omega=\sum\sigma_i\tau_i$$

where σ_i, τ_i are, respectively, differential forms of the first kind on $\boldsymbol{U}$, $\boldsymbol{V}$, of degree d_i, $r-d_i$. Moreover, if ω, $\boldsymbol{U}$ and $\boldsymbol{V}$ have a common field k of definition which is perfect, σ_i,τ_i are defined over k.

PROOF. Without loss of generality we may suppose that ω, $\boldsymbol{U}$ and $\boldsymbol{V}$ are defined over a perfect field k. Let $\boldsymbol{P}$ and $\boldsymbol{Q}$ be independent generic Points over k, of $\boldsymbol{U}$ and $\boldsymbol{V}$, respectively. If (t) and (u) are respectively, sets of uniformizing parameters at $\boldsymbol{P}$ and $\boldsymbol{Q}$, on $\boldsymbol{U}$ and $\boldsymbol{V}$, then

$$\begin{aligned}\omega&=\sum_{(i,j)}z_{i_1,\dots,i_s;j_1,\dots,i_{r-s}}dt_{i_1}\cdots dt_{i_s}du_{j_1}\cdots du_{j_{r-s}}\\&=\sum_j\Big(\sum_i z_{(i,j)}dt_{i_1}\cdots dt_{i_s}\Big)du_{j_1}\cdots du_{j_{r-s}}\end{aligned}$$

where $z_{i_1,\dots,i_s;j_1,\dots,j_{r-s}}$ are contained in $k(\boldsymbol{P},\boldsymbol{Q})$ and (i,j) means $i_1,\cdots,i_s;j_1,\cdots,j_{r-s}$. If we consider $\sum_i z_{(i,j)}dt_{i_1}\cdots dt_{i_s}$ as defined on $\boldsymbol{U}$ over the field $k(\boldsymbol{Q})$, they are of the first kind.

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1) Journal of the Mathematical Society of Japan Vol. 1, No. 3, 1949. This note will be denoted by $[K]$, and we shall use the same terminologies and notations as in $[K]$.