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A Generalization of Laguerre Geometry 1.

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§ 1. *Introduction.* In this paper we shall try to generalize the classical Laguerre differential geometry[1] in making use of the tensor calculus. Let V_n be an n-dimensional Riemannian space with the fundamental metric tensor $g_{ij}(x^k)$[2]. In each tangent Euclidean space referred to a cartesian coordinate system (X^i), a hypersphere is determined by the coordinates V^i of the center and its radius V^0, and is represented by an equation of the form

$$(1.1) \qquad g_{jk}(X^j-V^j)(X^k-V^k)=(V^0)^2.$$

The V^i are components of a covariant vector and V^0 is that of a scalar of V_n. A hypersphere will be hereafter denoted by the symbol V^λ[3] Thus each tangent space of V_n contains ∞^{n+1} hyperspheres. When it is regarded as the space whose elements are hyperspheres, we shall call it the *tangent space of hyperspheres*. Now, the tangential distance D between two hyperspheres V^λ and W^λ is given by

$$(1.2) \qquad D^2=g_{jk}(V^j-W^j)(V^k-W^k)-(V^0-W^0)^2$$

or by

$$(1.3) \qquad D^2=g_{\mu\nu}(V^\mu-W^\mu)(V^\nu-W^\nu),$$

where we have put

$$g_{00}=-1, \qquad g_{0k}=g_{k0}=0.$$

Now, a hyperplane in tangent space is represented by an equation of the form

$$(1.4) \qquad t_i X^i=p.$$

The necessary and sufficient condition that a hypersphere V^λ touches the hyperplane (1.4) is given by

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1) T. Takasu. Differentialgeometrieen in den Kugelräumen, Bd. 2, Laguerresche Differentialkugelgeometrie.

2) The indices i, j, k,······take the values 1, 2,······, n.

3) The indices λ, μ, ν,······, take the values 0, 1,···, n.