Journal of the Mathematical Society of Japan Vol. 2, Nos. 3-4 March, 1951.

On a Generalization of the Abe Groups.

Hiroshi UEHARA.

(Received July 23, 1949)

Introduction

In recent years various kinds of homotopy groups have been introduced as invariants of a topological space. Among others, M. Abe has introduced *a group* $\varkappa_n$, for every integer $n \geqq 2$, containing subgroups isomorphic to the n-dimensional homotopy group π_n and those isomorphic to the fundamental group π_1, in which the operation of π_1 on π_n forms an inner automorphism.

It is shown in this paper that this group $\varkappa_r$ can be extended in a certain way to obtain a *new group* $\sigma^{(r,n)}$, for $r \geqq n > 0$, containing a subgroup isomorphic to the r-dimensional homotopy group π_r. This group $\sigma^{(r,n)}$ will be called *the generalized Abe group of the type* (r, n). In a special case, this group corresponds with *the Abhomotopy group* $\varkappa^{(r,n)}$ introduced by S. T. Hu, which has a definite geometrical meaning: the homotopy classes consisting of mappings $f(S^r) \subset Z$ such that $f(S^n) = *$, where S^n denotes an n-dimensional subsphere of the r-dimensional sphere S^r, and $*$ is a base point of a topological space Z, constitute a group $\varkappa^{(r,n)}$ under a multiplication defined appropriately among them; while as was shown by M. Abe, $\varkappa_r$ is composed of the homotopy classes of mappings which transform S^r into Z and the subsphere S^0 (two points) into the base point $*$.

It has already been shown by Hu that the algebraic structure of the group $\varkappa^{(r,n)}$ is completely determined in terms of homotopy groups. My proof of this theorem is based upon Abe's arguments, and is simplified by utilization of the concept of the function space, which consists of all the Z-valued functions of r-variables with certain conditions.

In the latter part of this paper, I shall discuss several relations between the Abhomotopy group and the torus homotopy group; (i) the isomorphic imbedding of $\varkappa^{(r,n)}$ in *the r-dimensional torus homotopy group* τ_r, which was recently introduced by R. H. Fox[7] ii) the simplicity in the sense of Eilenberg.

I have to add here that a considerable part of the results in this paper happens to be duplicate to the results of S. T. Hu.[6] But my method is quite different from his.