Journal of the Mathematical Society of Japan Vol. 2, Nos. 3-4, March, 1951.

On Invariant Differential Forms on Group Varieties

Shigeo NAKANO.

(Received Apr. 17, 1950)

In this note we shall discuss the invariant differential foims on group varieties[1] and prove that for any group variety, there corresponds to it a Lie ring composed of invariant dervations of the (abstract) field of functions defined on that group variety. We shall also discuss some of its properties, which are the analogues of the case of usual Lie groups.

§ 1. Differential Forms on an Algebraic Variety.

Let V^n be a variety in S^N, $\mathfrak{D}_1(V)$ the totality of functions defined on $V\times V$ which induce on Δ_V the constant 0. $\mathfrak{D}_1(V)$ is a module over the field of constants Ω[2]. Let $\theta \in \mathfrak{D}_1(V)$ and let k be a field of definition for θ, P a generic point of V over k and $H_1(X),\ldots\ldots,H_n(X)$ a uniformizing set of linear forms of V at P. We shall denote by Λ_j^{2N-n+1} the linear variety in $S^N\times S^N$ defined by $H_i(X-X')=0$ $(i=1,\ldots\ldots,\hat{j},\ldots\ldots,n)$ (here $\hat{j}$ means to omit j). Then by F-VI_1 th. 1[3], $V\times V\cap\Lambda_j$ has a unique proper component W_j^{n+1} containing Δ_V, W_j has the multiplicity 1 in this intersection and Δ_V is simple on W_j. If, therefore, the function θ_{W_j} induced by θ on W_j is not the conatant 0, $(\theta)\cdot W_j$ is defined and we have

$$v_{\Delta_V}(\theta_{W_j})=\text{coeff. of } \Delta_V \text{ in } (\theta)\cdot W_j \geqq 1.$$

Proposition 1. *Let* $H_i'(X)$ $(i=1,\ldots\ldots,n)$ *be another uniformizing set of linear forms of* V *at* P *and* W_j' *be defined from* $H_i'(X)$ *as* W_j *were from* $H_i(X)$. *If for some* j $(1\leqq j\leqq n)$, θ_{W_j} *is not the constant* 0 *and* $v_{\Delta_V}(\theta_{W_j})=1$, *then the same is true for some* $\theta_{W_l'}$ $(1\leqq l\leqq n)$.

Proof. Let $P=(x)$, and $Q=(x')$ be a generic point of V over $k(x)$, then $P\times Q$ is a generic point of $V\times V$ over k. As θ is in the specialization ring of Δ_V in $k(x, x')$,

$$\theta(x, x')=\frac{f(x, x')}{g(x, x')}$$

where $f(X, X')$, $g(X, X')\in k[X, X']$ and $g(x, x)\neq 0$.
Since we are concerned with the components containing Δ_V, it does not matter whether we consider the function θ or f. If we consider the function F on $S^N\times S^N$ defined by $F(\bar{x}, \bar{x}')=f(\bar{x}, \bar{x}')$ where $(\bar{x}),(\bar{x}')$ are in-