Journal of the Mathematical Society of Japan Vol. 2 · Nos. 3–4, March 1951.

Conformal Representation of Multiply Connected Domain on Many-sheeted Disc.

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(Received Oct. 1, 1949)

Bieberbach[1] proved first, that any schlicht domain D bounded by p continua can be conformally mapped on a p-sheeted unit disc, i. e. on a Riemann surface, which covers each point inside the unit circle exactly p-times. Afterwards Grunsky[2] gave another proof of this theorem and added that this mapping is uniquely determinate under the condition mentioned later in Theorem 1.

In this paper we shall treat more generally the conformal mapping of D on a k-sheeted unit disc, where k is any integer $\geqq p$, and prove some results concerning this mapping. But since the general case $k \geqq p$ can be discussed quite the same, we will, for simplicity, confine ourselves to *the case* $k=p$ for a while.

Without loss of generality we can assume that D lies in the finite part of the x-plane, and is bounded by p closed analytic Jordan curves $\Gamma_1, \ldots\ldots, \Gamma_p$. We denote by $g(x, x')$ the Green's function of D with x' as its pole, and by $h(x, x')$ the conjugate function of $g(x, x')$. For $\lambda=1, \ldots\ldots, p$, let $u_\lambda(x)$ be the harmonic measure of the boundary curve Γ_λ with respect to D, and $w_\lambda(x)=u_\lambda(x)+iv_\lambda(x)$ be the regular function with the real part $u_\lambda(x)$. $w_\lambda(x)$ is regular on the closure $\bar{D}$ of D, and, if $p>1$, infinitely many-valued.

We use the following two lemmas several times in the sequel.

Lemma 1. *Let $x_1, \ldots\ldots, x_p$ be p points in D. If these points satisfy the p equations:*

$$\sum_{\mu=1}^{p} u_\lambda(x_\mu)=1 \quad (\lambda=1, \ldots\ldots, p), \tag{1}$$

and only in such a case, there exists a function $y=y(x)$, which maps D conformally on a p-sheeted unit disc Δ on y-plane, so that the images of $x_1 \ldots \ldots x_p$ on Δ have one and the same projection $y=0$. In this case, the mapping function is given by