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On Conformal Representation of Multiply Connected Polygonal Domain.

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It is known, that a function $w(z)$ is schlicht and star-shaped with respect to $w(0)=0$ in $|z|<1$, when, and only when, it can be expressed in the form

$$w(z)=\text{const.}\cdot z\cdot \exp.\ 2\int_{|\zeta|=1}\log\frac{\zeta}{\zeta-z}\,d\mu(\zeta),$$

where μ denotes a positive distribution of total mass 1 on the unit circle. This formula can also be written in the form

$$w(z)=\text{const.}\ \exp.\int_{|\zeta|=1}\log\frac{z}{\left(1-\frac{z}{\zeta}\right)^2}\,d\mu(\zeta),$$

and here comes out *Koebe's extremal function.* The argument of this function is equal to a constant on $|z|=1$ except the point ζ, and jumps by $+2\pi$ when z passes ζ in positive direction on $|z|=1$. Then, the above formula shows: The star-shaped function $w(z)$, whose argument is non-decreasing for z moving on $|z|=1$ in positive direction, can be constructed *from such elements as a sort of geometrical mean.*

We shall prove in this paper an analogue of this fact for n-ply connected domain, and, as an application thereof, treat the conformal representation of n-ply connected polygonal domain.

In order to simplify the wording, we call a half straight-line $\operatorname{Arg}\Omega=\text{const.}$, $|\Omega|\geqq \text{const.}>0$ an "*infinite radial slit*", and a segment $\operatorname{Arg}\Omega=\text{const.}$, $\text{const.}\geqq|\Omega|\geqq\text{const.}>0$ a "*radial slit*", respectively.

§ 1.

Let D be a domain on z-plane bounded by n analytic closed curves $\Gamma_1,\cdots\cdots,\Gamma_n$, whose sum we denote by Γ, and let z_0 be a fixed point in D.

For any point ζ on Γ, we denote by $\Omega(z,\zeta)$ the function which satisfies the conditions $\Omega(z_0,\zeta)=0$, $\Omega'(z_0,\zeta)=1$ and maps D conformally on the whole Ω-plane cut along an infinite radial slit and $(n-1)$ radial slits, so that the boundary point ζ of D corresponds to the bodunary point