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On the finite group with a complete partition

Michio SUZUKI.

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A *partition* of a group G is a system $\{H_i\}$ of subgroups of G such that every element of G except the unit element is contained in one and only one of the groups H_i. H_i are called components of this partition. A partion of G is called *complete*, when all of its components are cyclic. A tigroup with a complete partition is called *completely decomposable* (c. d.).

Of course not every group has a complete partition. In this paper we shall deal with finite groups with a complete partition, and determine the structure of such groups, when they are non-simple. Our main theorem is the following:

Let G be a non-simple, non-solvable c.d. group. Then G is isomorphic to the full linear fractional group of one variable over a finite field whose characteristic is greater than 2.

The author has, however, not yet been able to determine the structure of c.d. simple groups. Well-known simple groups $LF(2,p^n)$ are clearly c.d., and it is conjectured that no other c.d. simple group exists. Every known simple group contains one of $LF(2,p^n)$ as its subgroup, so $LF(2, p^n)$ may be regarded as the "least" simple group. It is suggested by this fact, as it seems to the author, that the problem to find the structure of c.d. simple groups would be an interesting and important one.

Finite groups with complete partitions have been considered by Kontorovitch [1][1] and [2]. His results will be sharpened to theorems 1, 2 and 3 of this paper and will play fundamental role in our study. This paper is written, so as to be read without reference to Kontorovitch, so that the results of §1 of this paper are essencially the same with his. In § 2 we shall determine the structure of c.d. solvable groups, and give the complete classification of such groups. In § 3 we shall give some remarks on the structure of c.d. groups and shall prove in § 4 the main theorem stated above. Our proof of this theorem is based on a characterization of linear

(1) The numbers in brackets refer to the bibliography at the end of the paper.