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On Continuous Geometries, II.

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In Part I of this paper[1] we have introduced a dimension function with values in a conditionally complete lattice-group, into an arbitrarily given continuous geometry, and imbedded the geometry into the direct sum of irreducible ones. We have proved, thereby, that the dimension is restrictedly additive[2], whence follows immediately the unrestricted additivity of perspectivity[3] This latter additivity had been already proved, however, as we were informed of after the publication of part I by I. Halperin[4]. In the following lines we shall show that the former additivity can be deduced easily from the latter (as was remarked in Part I). Also we shall give a new proof to Halperin's theorem of superposition of decompositions as an application of our theory.

All this will be done in generalizing the method of Part I in a certain sense. We shall namely show to what extent our previous method can be applied to obtain a generalized dimension function and the imbedding theorem, when we replace the perspective relation by an equivalence relation with some natural restrictions. In particular, it should be an extension of perspectivity, that is, any two elements should be in this relation if they are perspective. An example of such extension is that induced by a group of automorphisms of the geometry, considered by Halperin[5] and F. Maeda[6]. In this specified case, our restrictions are stronger than Maeda's, and weaker than Halperin's, and our dimension function can be obtained from Maeda's by means of the representation of a conditionally complete lattice-group by real-valued continuous functions. But, this being concerned only with the dimension function, the subject of this note may, as we hope, appeal to wider interest.

§ **I.** This section is devoted to some preparatory considerations about a conditionally complete, and so abelian, lattice-group, which may be of some interest in themselves, The letter $\mathfrak{G}$ will denote throughout this paper, unless otherwise qualified, always such a group and $f, g, h, \ldots\ldots$ its elements. These letters will be used with or without indices. If we write f', we mean an element of such a lattice-group $\mathfrak{G}'$ with the above mentioned