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Conformal Mapping of Polygonal Domains.*

Yûsaku KOMATU.

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§ 1. Introduction.

It is well known that a function, which maps a circular disc or a half-plane onto the interior of a polygon, is given by the formula of Schwarz-Christoffel. Let $w=f(z)$ be such a function and let the image-polygon, laid on w-plane, have m vertices corresponding to the points a_μ $(\mu=1,\dots, m)$ on z-plane. Denoting by $\alpha_\mu\pi\,(0<\alpha_\mu<2)$ the interior angle at vertex $f(a_\mu)$, the Schwarz-Christoffel formula may be written as follows:

$$f(z)=C\int^{z}\prod_{\mu=1}^{m}(a_\mu-z)^{\alpha_\mu-1}dz+C', \tag{1·1}$$

where C and C' are both constants depending on position and magnitude of image-polygon.

The present author has previously shown that this formula can be generalized to the case of analogous mapping of doubly-connected domains.[1] We may take, as a standard doubly-connected basic domain, an annular domain $q<|z|<1$, $-\lg q$ being a uniquely determined conformal invariant, i. e. the so-called *modulus* (*Modul*) of given polygonal domain. Let the boundary components corresponding to circumferences $|z|=1$ and $|z|=q$ be polygons with m and n vertices respectively. Let further $\alpha_\mu\pi$ and $\beta_\nu\pi$ denote the interior angles (with respect to each boundary polygon itself) at vertices $f(e^{i\varphi_\mu})$ and $f(qe^{i\psi_\nu})$ respectively. The mapping function $w=f(z)$ is then given by the formula

*) A preliminary report under the same title has been published in Kōdai Math. Sem. Rep. Nos. 3–4 (1949), 47250.

1) Y. Komatu, Darstellungen der in einem Kreisringe analytischen Funktionen nebst den Anwendungen auf konforme Abbildung über Polygonalringgebiete. Jap. Journ. Math. **19** (1945), 203–215.