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On a conformal mapping with certain boundary correspondences.

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Given any set of points E on the unit circle C of z-plane, we shall treat in this paper the problem to map the interior of C conformally on a schlicht domain D so that the set E corresponds to a set of accessible boundary points of D all lying on one and the same point of the plane.

In order to simplify the wording, we call a half straight-line on w-plane: arg w=const., $\infty \geqq |w| \geqq$ const. >0, an infinite radial slit.

First, we consider the case where the set E is *finite*.

Theorem 1. *Let $z_1,\dots,z_n$ be n points on C associated with n positive numbers $a_1,\dots,a_n$, whose sum is equal to 1. Then, there exists a function $w=w(z)$, which maps the interior of C conformally on a domain D, so that:* 1. *D is the whole w-plane cut along n infinite radial slits,* 2. *z_k corresponds to the accessible boundary point of D lying on $w=\infty$, which is determined by an angular domain between two of these slits including an angle $2\pi a_k$ at $w=\infty$, and* 3. *$w(0)=0$, $w'(0)=1$. Under these conditions the mapping function is uniquely determined and is given by*

$$w=w(z)=z\Big/\prod_{k=1}^{n}\left(1-\frac{z}{z_k}\right)^{2a_k}. \tag{1}$$

Proof. We construct a potential function $u(z)$ on z-plane, whose singularities are

$$\log|z| \quad \text{at} \quad z=0,$$

$$\log\frac{1}{|z|} \quad \text{at} \quad z=\infty,$$

and

$$2a_k \log\frac{1}{|z-z_k|} \quad \text{at} \quad z=z_k.$$

Denoting the conjugate potential of $u(z)$ by $v(z)$, we put

$$w(z)=\text{const.}\ \exp.\ \{u(z)+iv(z)\}.$$