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Riemann Spaces of Class two and their Algebraic Characterization

Part III

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In the two preceding papers (part 1 and II)[1] we have defined the type number of $n(\geqq 4)$-dimensional Riemann space R_n and, making use of it, have got a necessary and sufficient condition that there be a set of functions H^{I}_{ij} and H^{II}_{ij} satisfying the Gauss equation

$$R_{ijkl}=H^{P}_{ik}H^{P}_{jl}-H^{P}_{il}H^{P}_{jk} \quad (P=I,\ II;\ i,\ j,\ k,\ l=1,\ldots\ldots,\ n),$$

when $R_n(n\geqq 6)$ is of type $\geqq 3$. And then we have had the theorem 4.4 of the part II, i. e. a necessary and sufficient condition that $R_n(n\geqq 8)$ of type $\geqq 4$ be of class two, making use of the theorem 1.5 of the part I.

In this part III, we conider the Codazzi and Ricci equations when $R_n(n\geqq 6)$ is of type $\geqq 3$, and get a necessary and sufficient condition that $R_n(n\geqq 6)$ of type *three* be of class two.

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§ 1. Introduction

In § 1 of Part I we put

$$L_{ijkl}=H^{I}_{ij}H^{II}_{kl}-H^{I}_{il}H^{II}_{jk}-H^{II}_{ij}H^{I}_{kl}+H^{II}_{il}H^{I}_{jk}, \tag{1·1}$$

and

$$K^{\cdot P}_{Q\cdot ij}=g^{cd}(H^{Q}_{ci}H^{P}_{dj}-H^{Q}_{cj}H^{P}_{di}). \tag{1·2}$$

If we put $K_{ij}=K^{I}_{II,ij}$, we have