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Riemann Spaces of Class Two and Their Algebraic Characterization.

Part II.

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In this paper we give a necessary and sufficient condition that a Riemann space $R_n (n \geqq 8)$ be of class two, making use of the type number discussed in a preceding paper[1].

§ 1. A reality condition

Suppose that a Riemann space $R_n (n \geqq 6)$ of class two is of type $\geqq 3$, and put

$$K_{ij} = p_{ij} + e q_{ij} \quad (e^2 = -1); \tag{1·1}$$

where the tensor K_{ij} is the solution of (1·9) of the part I, i. e.

$$M_{ijkl} = K_{i(j} K_{kl)}, \tag{1·2}$$

and p's and q's are all real. The tensor M_{ijkl} in (1·2) is defined by (1·10) of the part I, i. e.

$$M_{ijkl} = \frac{1}{2} (R_{c\cdot i(j}^{\ a} R_{|a|\cdot kl)}^{\ c}). \tag{1·3}$$

Substituting (1·1) in (1·2) and equating to zero the imaginary parts we have

$$p_{i(j} q_{kl)} + q_{i(j} p_{kl)} = 0. \tag{1·4}$$

(A) Suppose that det. $|q| \neq 0$. Contracting (1·4) by q^{kl} we have

$$(n-4) \ p_{ij} + q^{ab} p_{ab} q_{ij} = 0,$$

and contracting it by q^{ij} we have $q^{ab} p_{ab} = 0$ for $n > 2$. Therefore all of p_{ij} are zero for $n \geqq 6$. Hence the K's are pure imaginary except zero.

(B) Suppose that det. $|q| = 0$. If the rank of $\|q\|$ is $2\sigma (n > 2\sigma \geqq 6)$, we have similarly $p_{ij} = 0$ for $i, j = 1, \ldots\ldots, 2\sigma$.

Next putting $k, l = 1, \ldots\ldots, 2\sigma$ and $i, j > 2\sigma$ in (1·4) we have $q_{kl} p_{ij} = 0$,