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Isoperimetric Inequalities.

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1. The most simple isoperimetric problem formulated by Steiner[1] may be stated as follows: *Among all rectifiable closed Jordan curves on a plane with assigned length, determine the one which maximizes the enclosing area.*

The solution of the problem is, as is well known, given by *circle*; in other words, *if F denotes the area of any (finite) domain bounded by a rectifiable closed Jordan curve with length L, then the isoperimetric inequality*

$$(1) \qquad 4\pi F \leqq L^2$$

always holds, and the equality sign appears when and only when the surrounding curve is a circle.

Beside a purely geometrical proof due to Edler[2], an elegant analytical proof of this fact has been given by Hurwitz[3]. But Hurwitz, making use of Fourier series, assumed in his proof the piecewise smoothness of boundary curves in order to confirm the termwise differentiability of the series. And then various generalizations and brief proofs of the proposition have been published by Brunn[4], Minkowski[5], Carathéodory-Study[6] and others.

On the other hand, Bieberbach[7] has shown an analogous isoperimetric

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1) J. Steiner, Einfache Beweise der isoperimetrischen Hauptsätze. Journ. reine u. angew. Math. **18**, (1838), 289–296. Cf. throughout this Note as a reference the excellent report by T. Bonnesen u. W. Fenchel, Theorie der konvexen Körper. (Ergebn. d. Math. III 1.) Berlin (1934).

2) F. Edler, Vervollständigung der Steinerschen elementargeometrischen Beweise für den Satz, dass der Kreis grösseren Flächeninhalt besitzt als jede andere Figur gleich grossen Umfangs. Nachr. Ges. Wiss. Göttingen (1882), 73–80.

3) A. Hurwitz, Sur le problème des isopérimètres. C. R. Acad. Sci. Paris **132** (1901), 401–403; Sur quelques applications géométriques des séries de Fourier. Ann. École Norm. Sup. (3) **19** (1902), 357–408.

4) H. Brunn, Über Ovale und Eiflächen. Inaug. Diss. München (1887), 42 S.

5) H. Minkowski, Allgemeine Lehrsätze über konvexe Polyeder. Nachr. Ges. Wiss. Göttingen (1897), 198–219; Volumen und Oberfläche. Math. Annalen **57** (1903), 447–495.

6) C. Carathéodory u. E. Study, Zwei Beweise des Satzes, dass der Kreis unter allen Figuren gleichen Umfangs den grössten Inhalt hat. Math. Annalen **68** (1909), 133–140.

7) L. Bieberbach, Über eine Extremaleigenschaft des Kreises. Jahresb. Deutsch. Math-Verein. **24** (1915), 247–250.