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On the dimension of normal spaces. II.

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In the present paper we shall generalize some of the results obtained in a previous paper [5].[1]

As is well known, a normal space R is called to be of dimension not greater than n, dim $R \leqq n$, in case for any finite open covering of R there exists an open refinement of order not greater than $n+1$. Our main theorem reads as follows: Let $\{G_\alpha;\ \alpha \in \Omega\}$ be a locally finite system of open sets in a normal space R and $\{F_\alpha;\ \alpha \in \Omega\}$ a system of closed sets such that $F_\alpha \subset G_\alpha$, $\alpha \in \Omega$. If the dimension of a closed set A of R is not greater than n, then there exists a system $\{U_\alpha;\ \alpha \in \Omega\}$ of open sets in R such that (1) $F_\alpha \subset U_\alpha \subset G_\alpha$ $\alpha \in \Omega$ and (2) the order of the system $\{A\cdot(\bar{U}_\alpha - U_\alpha);\ \alpha \in \Omega\}$ is not greater than n.

As an application of this theorem, we can prove a theorem that for a metrizable space R the relation dim $R \leqq n$ implies the relation $\dim^* R \leqq n$, where we mean by $\dim^* R$ the dimension of R in the sense of Menger-Urysohn. In particular, for the case that R is a metric space with the star-finite property, the relation dim $R \leqq n$ is shown to be equivalent to $\dim^* R \leqq n$. This theorem may be considered as a generalization of a well-known theorem for separable metric spaces, since such spaces have necessarily the star-finite property (Cf. [6]).

Besides the results mentioned above some other theorems will also be obtained.

§ 1. Locally finite systems.[2]

A system $\mathfrak{W}$ of subsets in a topological space R is called to be locally finite, if for each point p of R there exists a neighbourhood $U(p)$ such that $U(p)$ intersects a finite number of sets of $\mathfrak{W}$.

Theorem 1.1. *Let* $\{G_\alpha;\ \alpha \in \Omega\}$ *be a locally finite open covering of a*

1) Numbers in brackets refer to the Bibliography at the end of the paper.

2) The results of §§ 1,2 and 3 were published in [7] except Theorems 2.4 and 3.2.