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On affine collineations in projectively related spaces.

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§1. **Introduction.** M. S. Knebelman[1] proved a theorem on motions in conformally related Riemannian spaces: If an n-dimensional Riemannian space V_n admits an r-parameter group G_r of motions $(r<n)$, then there exist $n-r$ independent Riemannian spaces which are conformal to the Riemannian space V_n and admit the group G_r as group of motions.

One of the present authors has given in his forthcoming book a simple proof of this theorem. Let X_a be the r infinitesimal operators of the group G_r of motions in V_n whose fundamental tensor is $g_{\mu\nu}$, then we have

$$X_a g_{\mu\nu}=0, \qquad (a,b,c,\ldots=1,2,\ldots r\,;\ \lambda,\mu,\nu,\ldots=1,2,\ldots,n). \tag{1.1}$$

In order that a Riemannian space which is conformal to V_n and consequently whose fundamental tensor is of the form $\rho^2 g_{\mu\nu}$ admit the G_r as a group of motions, it is necessary and sufficient that we have

$$X_a(\rho^2 g_{\mu\nu}) \doteq 0,$$

from which,

$$X_a \rho^2=0, \tag{1.2}$$

because of (1.1). But, X_a being the operators of a group, that is, X_a satisfying the relations

$$(X_b X_c)f=c_{bc}^{\cdot\cdot a} X_a f, \tag{1.3}$$

the equations (1.2) are completely integrable. Thus the theorem of Knebelman is proved.

The purpose of the present Note is to give a simple proof of an analogous theorem for group of affine collineations in projectively related spaces which is also due to Knebelman.[3]

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1) M. S. Knebelman: On groups of motions in related spaces. Amer. Jour. of Math., 52(1930), 280-282.

2) K. Yano: Groups of transformations in generalized spaces, in press.

3) M. S. Knebelman: Collineations of projectively related affine connections. Annals of Math., 29 (1928), 389—394.