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On removable singularities of an analytic function of several complex variables.

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1. Let $u(P)=u(x_1,\dots\dots\dots, x_n)$ be defined in a domain D in an n-dimensional space and all its partial derivatives of the second order be continuous and satisfy the equation :

$$\Delta u=\frac{\partial^2 u}{\partial x_1^2}+\cdots+\frac{\partial^2 u}{\partial x_n^2}=0, \tag{1}$$

then $u(P)$ is called a harmonic function in D. It is easily seen that $u(P)=\overline{OP}^{n-2}$ $(n\geqq 3)$ is a harmonic function, where P is a variable point and O is a fixed point.

Let $\sum$ be a sphere in an n-dimensional space with O as its center and R be its radius and S be its boundary. Let Q be a point of S and $\varphi(Q)$ be an integrable function on S. We define a Poisson integral with $\varphi(Q)$ as its boundary value :[1]

$$u(P)=\frac{1}{RS_n}\int_S \varphi(Q)\,\frac{R^2-\overline{OP}^2}{\overline{PQ}^n}\,d\sigma_Q, \tag{2}$$

where S_n is the surface area of a unit sphere and $d\sigma_Q$ is the surface element of S at Q. Then $u(P)$ is harmonic in $\sum$.

We can prove that $u(P)$ tends to $\varphi(Q)$ almost everywhere on S, when P tends to Q non-tangentially to S. If $\varphi(Q)$ is continuous at Q_0, then $u(P)$ tends to $\varphi(Q_0)$, when P tends to Q_0 from the inside of $\sum$. Let $u(P)$ be a bounded harmonic function in $\sum$, then $\lim u(P)=\varphi(Q)$ exists almost everywhere on S, when P tends to Q non-tangentially to S and $u(P)$ can be expressed by (2)[2].

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1) For an n-dimensional Poisson integral, c. f. C. Carathéodory: On Dirichlet problem. Amer. Jour. Math. **59**(1937).

2) The case n=2 is the well known theorems of Fatou and Schwarz. For the case n=3 I have proved analogous theorems in a paper: On Fatou's theorem on Poisson integrals. Jap. Jour-Math. **15**(1938). The method can be applied for the general case.