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On the decomposition of an (L)-group

YOZÔ MATSUSHIMA

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The purpose of the present note is to give a decomposition theorem of an (L)-group[1] which is a generalization of the well known theorem of Levi[2] in the theory of Lie groups.

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§1. A locally compact group G is called an *(L)-group*, if G contains a system of closed normal subgroups $\{N_\alpha\}$ such that

1) G/N_α are all Lie groups and
2) $\bigcap_\alpha N_\alpha = e$,

where e denotes the unit element of G. If an (L)-group G is connected, G contains a system of compact normal subgroups $\{K_\alpha\}$ such that G/K_α are all Lie groups and $\bigcap_\alpha K_\alpha=e$. Moreover we may assume that all K_α are contained in a compact normal subgroup and that the intersection of any finite number of K_α is contained in the system $\{K_\alpha\}$[3]. Such a system $\{K_\alpha\}$ is denoted in the following as a *canonical system* of G.

A subgroup L of an (L)-group G is called a Lie subgroup, if it is generated by a local Lie group L_l which is contained in a neighbourhood of the unit element of G. If we take as the neighbourhoods of the unit element in the group L those of the local group L_l we may introduce a new topology in L, which we shall call the inner topology of L.

Now let G be an arbitrary topological group and let H_1 and H_2 be the subgroups of G. We denote by $[H_1, H_2]$ the subgroup of G generated by the elements of the form $[h_1, h_2]=h_1 h_2 h_1^{-1} h_2^{-1}$. The closure of $[H_1, H_2]$ will be denoted by $C(H_1, H_2)$ and is called the topological commutator group of H_1 and H_2. In particular $C(G, G)$ is called the topological commutator group of G and is denoted by $D(G)$. We define inductively the groups $D_n(G)$ by the relations $D_0(G)=G$, $D_n(G)=D(D_{n-1}(G))$. Analogously the subgroups $N_n(G)$ are defined by $N_0(G)=G$. $N_n(G)=C(G, N_{n-1}(G))$. A connected locally compact group G is *solvable* (*nilpotent*), if $D_n(G)=e$ ($N_n(G)=e$) for some integer n. In the case of the Lie groups these definitions of the solvability and the nilpotency coincide with the usual ones.