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On Projectively Connected Spaces whose Groups of Holonomy Fix a Hyperquadric

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Introduction. This paper deals with n-dimensional projectively connected spaces P_n whose groups of holonomy fix a hyperquadric. For the spaces with normal connexion, similar results as ours have been obtained independently by S. Sasaki and K. Yano.[1)]

In § 1, 2, we shall arrive at the fundamental equations (22) and (24) in the case of the spaces with no torsion, following the general method of E. Cartan in his famous paper.[2)] In § 3, we shall consider the Riemannian space R^*_{n+1} which is associated with the space P_n and obtain a relation between these two spaces which shows that the condition in order that the connexion of P_n be normal is equivalent to that of R^*_{n+1} being an Einstein space. In § 4, we shall investigate the relations between the space P_n and the Riemannian space $R_n{}^*$ which is a hypersurface in R^*_{n+1}. Then, in § 5, we shall show that there exist a Riemannian space R_n which is projective to P_n and that, if the connexion of P_n is normal, R_n is an Einstein space. Lastly, in § 6, we shall show that R_n is the space treated by E. Cartan.[3)]

§ 1. According to E. Cartan, let $R:(A,\ A_i)$ $(i=1,2,\ldots\ldots n)$ be a frame of an n-dimensional space P_n with projective connexion. Then the connexion is given by a system of Pfaffians $\omega^\lambda_\mu(\lambda,\ \mu=0,1,2,\ldots\ldots,n)$ such that

$$(1)\qquad dA=\omega^0_0 A+\omega^i A_i,\qquad dA_i=\omega^0_i A+\omega^j_i\ A_j$$

where $\omega^i=\omega^i_0$. The equations of structure of P_n are

$$(2)\qquad (\omega^\mu_\lambda)'=[\omega^\rho_\lambda\ \omega^\mu_\rho]-\Omega^\mu_\lambda,$$

$$\Omega^\mu_\lambda-\delta^\mu_\lambda\Omega^0_0=\frac{1}{2}A_\lambda{}^\mu{}_{ij}\ [\omega^i\omega^j]$$

where $A_\lambda{}^\mu{}_{ij}$ are the components of the tensor of curvature and torsion of the space. In a coordinate neighborhood (y^i), we can use natural frames such that the following relations hold:

$$(3)\qquad \omega^i=dy^i,\qquad \omega^i_i-n\omega^0_0=0.$$

Let us now represent a non degenerate hyperquadric Q_{n-1}, which the