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Base conditions for hypersurfaces at a point.

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In this paper we shall study systematically the base condition at a given point for hypersurfaces in an n-dimensional affine space over an arbitrary ground field K.

A base condition for a system of hypersurfaces will be expressed by a certain set of linear (homogeneous) relations between the coefficients of the equations of hypersurfaces belonging to the system. Namely, taking the given point O as the origin of the coordinate system $OX_1X_2\dots X_n$, and the equation of such hypersurface being $f=\sum a_{ij\dots k}X_1^iX_2^j\dots X_n^k=0$ (a' $s \in K$), the base condition is expressed by a set of equations

$$\sum a_{ij\dots k}a_{ij\dots k}^{(\lambda)}=0 \quad (a\text{' } s \in K) \quad (\lambda=1, 2,\dots\dots)$$

for the coefficients of the polynomial f. The totality of polynomials satisfying the base condition forms an ideal in the ring of polynomials.

Since the degree of the polynomial is not assigned by base conditions, it is preferable to deal more generally with formal power series. In §§ 2—3 the base condition will be discussed as a set of linear conditions related with linear mappings of K-vector space into K. In § 4 we shall characterize the base condition by using the Macaulay's inverse system from a new point of view.[1] In § 5 some results concerning with irreducible ideals are obtained.

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1. *The ring of formal power series.*

Let L be the ring $K[[X_1,\dots\dots,X_n]]$ of formal power series in $X_1,\dots\dots,X_n$ over K. Let us arrange all non-negative power products of $X_1,\dots\dots, X_n$ lexicographically and consider them linearly ordered. We denote them, for brevity, by $x_i (i=1, 2,\dots\dots)$. Then, any series f of L is expressible in the form $f=\sum_{i=1}^{\infty} a_i x_i$ ($a_i \in K$). If $a_1=\dots\dots=a_{r-1}=0$, $a_r\neq 0$, then r will be called the *rank* of f.

Let $f=\sum_{i=1}^{\infty} a_i x_i$ be a series of L. We introduce in L a weak topology, namely we define a neighborhood of f, for each finite set $i_1,\dots\dots,i_m$ of positive integers, as being the set of all such series $\sum_{i=1}^{\infty} a'_i x_i$ that a'_i is equal