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On the jump of a function and its Fourier series. Notes on Fourier Analysis (XXXIII)

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§ **1.** Let $f(x)$ be an integrable and periodic function with period 2π and its Fourier series be

$$\mathfrak{S}[f]=\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n \text{ con } nx+b_n \sin nx).$$

Fejér has proved that, if there is an s such that

$$\int_0^1|\psi(u)-s|du=o(t),$$

$$\psi(u)=(f(u)-f(-u))/2,$$

then the sequence (nb_n) is $(R, \log n, 1)$-summable to $2s/\pi$.

Recently, O. Szàsz has proved that if

$$\int_0^t(\psi(u)-s)\,du=o(t) \tag{1}$$

and

$$\int_0^t|\psi(u)-s|du=O(t), \tag{2}$$

then the sequence (nb_n) is $(C, 2)$-summable to $2s/\pi$.

We shall now consider the $(R, \log n, \alpha)$-summability of the sequence (nb_n). In fact we shall prove the following theorems:

Theorem 1. *If for any* $\alpha\geqq 0$

$$\lim_{t\to 0}\psi(t)=s \quad (R, \log n, \alpha),$$

then the sequence (nb_n) *is* $(R, \log n, 1+\alpha+\delta)$*-summable to* $2s/\pi$, *where* δ *is any positive number.*

Theorem. 2. *If for any* $\alpha\geqq 1$, (bn) *is* $(R, \log n, \alpha)$ *— summable to* $2s/\pi$, *then*

$$\lim_{t\to 0}\psi(t)=s \quad (R, \log n, \alpha+1+\delta),$$

δ *being any positive number.*

§ **2.** **Lemmas.** Let us put

$$l_\alpha(t)=\frac{1}{t}\int_0^t(\log(t/u))^\alpha \sin u\,du$$