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On some properties of covering groups of a topological group.

Yukiyosi KAWADA

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Recently C. Chevalley has developed in his book "Theory of Lie groups" (cited with L.G.) the theory of covering groups of a connected, locally connected and locally simply connected topological group with new definitions of a covering space and of the simply connectedness of a space.

The purpose of this paper is to investigate some properties of covering groups of a topological group with these new conceptions. In § 1 we shall give an algebraic characterization of the simply-connectedness of a topological group and give another proof of the existence theorem of a simply-connected covering group under usual conditions. In § 2, § 3 we shall consider the generalized universal covering group under weaker conditions, i.e. for a connected, locally connected topological group with the first countability axiom.

§ 1. *Simply connected topological groups.*

We use here the following definitions from L.G., the definition that a set $E \subseteqq X$ is evenly covered by X^* with respect to a continuous mapping f of X^* into X (Chap. II, § VI, Def. 2); the definition of a covering space (X^*, f) of X, where X and X^* are connected (conn.) and locally connected (l.c.) space with a continuous mapping f of X^* onto X (§ VI, Def. 3) the definition of the simply connectedness of a conn. and l.c. space (§ IX, Def. 2); and the definition of a covering group of a conn. and l.c. topological group (§ VIII, Def. 2).

Definition. 1. Let G_1, G_2 be two topological groups and U_1 (U_2) be a neighbourhood of the unit element of G_1 (G_2) respectively. We mean by a *U_1-U_2-local isomorphism* of G_1 and G_2 a homeomorphism f of U_1 onto U_2 which has the following properties:

(i) the conditions $a,b,ab \in U_1$ imply $f(ab)=f(a)\,f(b)$ in U_2.

(ii) the conditions $a,b \in U_1$, $f(ab) \in U_2$ imply $ab \in U_1$.

Now we construct a topolngical group $Gr(U)$ from a neighbourhood U of the unit element e in a topological group G with the property $U=U^{-1}$ as follows. To each element a $(a\neq e)$ in U take an abstract element A. $Gr(U)$ has these $\{A\}$ as generators. If to a, a^{-1} $(a\neq e)$ in U correspond