Journal of the Mathematical Society of Japan Vol. 2, No. 3, Dec. 1949.

On the Differential Forms of the First Kind on Algebraic Varieties.

SHOJI KOIZUMI.

(Received April 10, 1949).

In his book "Foundations of algebraic geometry"[1] A. Weil proposed several problems concerning differential forms on algebraic varieties. In this note we shall take up some of them. Especially we shall discuss differential forms of the first kind which are defined on a complete abstract varieties without multiple point. Here the field of definition is assumed to be arbitrary.

1. Let $K=k(x_1,\ldots\ldots, x_n)=k(x)$ be a field, generated over a field k by a set (x) of quantities; the totality $\mathfrak{D}$ of all derivations in $k(x)$ over k forms a finite K-module. Every element z of K defines a linear function dz from $\mathfrak{D}$ into K; we call this linear function the differential of z, and we can define multiplication between a differential and an element of K as usual. The set $\mathfrak{F}$ of those linear functions, which are sums of the products thus obtained, forms the dual K-module of $\mathfrak{D}$, and therefore the dimensions of $\mathfrak{D}$ and $\mathfrak{F}$ are equal.

As usual we can form the Grassmann algebra from the finite K-module $\mathfrak{F}$. An homogeneous element, of degree m, is called a differential form of degree m, belonging to the extension $k(x)$ of k.

PROPOSITION 1. Let $K=k(x)$ be a separably generated extention of k, and $dim_k\ (x)=n$. If $(u_1,\ldots\ldots,u_n)$ is a set of elements of $k(x)$, such that $k(x)$, such that $k(x)$ is separably algebraic over $k(u)$, then every differential form belonging to the extention $k(x)$ of k can be expressed in one and only one way, as polynomials in $du_1,\ldots\ldots,du_n$ with coefficients in $k(x)$.

PROOF. Let z be an arbitrary element of K; it is sufficient to prove that dz is expressed uniquely as a linear form in $du_1,\ldots\ldots,du_n$ with coefficients in $k(x)$. As z is separably algebric over $k(u)$, there exists a polynomial $P(U, Z)$ in $k[U_1,\ldots\ldots,U_n, Z]$ such that $P(u, z)=0$, $P_Z(u,z)\neq 0$,

During my investigation I have received kind criticisms from Mr. Igusa to whom I express my hearty thanks.

1) In this note we shall stick throughout, in terminologies and notations, to Weil, l. c.