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An operator-theoretical treatment of temporally homogeneous Markoff process

KÔSAKU YOSIDA

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1. *Introduction.* Let $\{U_t\}$, $0 \leqq t < \infty$, be a one-parameter semi-group of linear (=everywhere defined additive, continuous) operators from a complex Banach space E to E:

(1.1) $$U_t U_s = U_{t+s}.\quad U_0 = I \quad (=\text{the identity operator}).$$

(1.2) $$\sup_t \| U_t \| \leqq 1,$$

(1.3) $$\lim_{t \to t_0} U_t x = U_{t_0} x,\ 0 \leqq t_0 < \infty \quad (\lim = \text{strong limit}).$$

In a preceding note[1], the author obtained the following results. i) If D is the totality of x for which

(1.4) $$\text{weak}\lim_{h \downarrow 0} h^{-1}(U_h - I)x = Ax$$

exists, then D coincides with the totality of x for which

(1.4)′ $$\lim_{h \downarrow 0} h^{-1}(U_h - I)x = Ax$$

exists and D is dense in E. The differential quotient operator (d.q.o.) A is a closed additive operator from D to E with the properties:

(1.5) $$U_t x - x = \int_0^t U_s Ax\, ds \quad \text{for } x \in D,$$

(1.6) for any positive integer n, $I_n = (I - n^{-1} A)^{-1}$ exists and $\| I_n \| \leqq 1$, $AI_n = n(I_n - I)$, $\lim_{n \to \infty} AI_n x = Ax$ for $x \in D$,

(1.7) $$I_n x = \int_0^\infty n \exp(-nt) U_t x\, dt \quad \text{and} \quad \lim_{n \to \infty} I_n x = x \quad \text{for } x \in E.$$

(1.8) $U_t x = \lim_{n \to \infty} \exp(tAI_n)\, x$, $x \in E$, uniformly in t for any finite interval of t.[2]

ii) Let conversely A be an additive operator from a dense linear subset D of E such that (1.6) is satisfied for any positive integer n, then there

1) On the differentiability and the representation of the one-parameter semi-group of linear operators, the Journal of the Math. Soc. of Japan, 1 (1948).

2) We may obtain, similarly as (1.8), another representation of U_t:

(1.8)′ $$U_t x = \lim_{n \to \infty} (I - n^{-1} t A)^{-n} x.$$