Journal of the Mathematical Sociecy of Japan Vol. 2, No. 3, Dec, 1949.

Determination of Function by its Fourier Series. Notes on Fourier Analysis (XII)

SHIN-ICHI IZUMI,

§ **1.** Introduction. Let $f(x)$ be an integrable function with period 2π, and $\bar{s}\ (x)$ and $\bar{\sigma}_n\ (x)$ be the $(n+1)$-th conjugate paitial sum and arithmetic mean of the Fourier series of $f(x)$, respectively. If x is a point of discontinuity of $f(t)$ of the first kind, we put

$$l(x)=f(x+0)-f(x-0).$$

Fejer[1] has proved that

(1) $$\lim_{n\to\infty} \bar{s}_n\ (x)\ /\log\ n=-l(x)/\pi.$$

Later Lukacs[2] proved that, if there is an $l(x)$, such that

$$\int_0^t |\ \psi_x(t)-l(x)\ |dt=o(t),$$

$$\psi_x(t)=f(x+t)-f(x-t),$$

as $t\to 0$, then (1) nolds. In this case x need not be the point of discontinuity of the first kind.

Recently 0. Szász[3] proved that, if there is an $l(x)$ snch that

(2) $$\int_0^t \{\psi_x(t)-l(x)\}dt=o(t)\ \bullet$$

and

(3) $$\int_0^t |\ \psi_x(t)-l(x)|\ dt=O(t),$$

then

(4) $$\lim_{n\to\infty} (\bar{\sigma}_{2n}(x)-\bar{\sigma}_n(x))=l(x).\ (\log\ 2)/\pi.$$

Mr. Matsuyama proposed the problem: do (2) and (3) imply (1)? This problem seems to be negative. In this paper we prove that (2) and

(5) $$\int_{\pi/n}^{\pi} \frac{|\psi_2(t)-\psi_x(t-\pi/n)|}{t}\ dt=o\ (\log\ n)$$

1) L. Fejér, ibidem, 142 (1913).

2) F. Lucács, Jour. für Math., 150 (1920).

3) O. Szász, Duke Math. Journ., 4 (1938).