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Galois theory for general rings with minimum condition.

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In a very suggestive paper N. Jacobson founded a Galois theory for division rings.[1] The theory was then skilfully extended by G. Azumaya to simple, and to uni-serial rings.[2] The present work is to establish it for general rings with minimum condition.[3] Most of our arguments are modifications or generalizations of theirs, while the others are those which have been employed in a previous note on semilinear repesentations and normal bases in noncommutative domains,[4] and we shall also resume the theorem of semilinear normal basis in a generalized and improved form.

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§ 1. Crossed product.

Let R be a ring with unit element 1 and satisfying the minimum condition (whence the maximum condition) for ideals. Let σ be an automorphism of R. For a two-sided module $\mathfrak{m}$ of R we can introduce a new two-sided module $(\mathfrak{m}, \sigma)$ of R which coincides with $\mathfrak{m}$ as right-module and for which left operation by R is defined: $a * u = a^{\sigma} u$ $(a \in R, u \in \mathfrak{m})$.

We call a finite group of automorphisms $\mathfrak{G} = \{1, \sigma, \ldots\ldots, \tau\}$ a *Galois group* of R when the following condition is satisfied:

(*) *(R, σ), (R, τ) with distinct σ, τ in $\mathfrak{G}$ have no isomorphic non-vanishing sub-residue-moduli.*

If $\mathfrak{b}$ is a $\mathfrak{G}$-invariant ideal in R, our Galois group $\mathfrak{G}$ of R can be, in natural manner, looked upon as that of the residue-ring $R/\mathfrak{b}$.

A crossed product (=semilinear group ring with factor set) of R with

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1) N. Jacobson, The fundamental theorem of Galois theory for quasi-fields, Ann. Math. **41** (1940).

2) G. Azumaya, New foundation for the theory of simple rings, forthcoming in Proc. Imp. Acad. Japan: G. Azumaya, Galois theory for uni-serial rings, Journ. Math. Soc. Japan 1 (1949).

3) Another extreme case is that of (closed) irreducible rings. See T. Nakayama, Note on irreducible rings, forthcoming in Proc. Imp. Acad. Japan; T. Nakayama-G. Azumaya, On irreducible rings, Ann. Math. 48 (1947).

4) T. Nakayama, Semilinear normal basis for quasifields, Amer. Journ. Math. 71 (1949).