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On the Algebraic geometry of Chevalley and Weil

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During the War CHEVALLEY and WEIL have developed independently two theories of intersections of algebraic varieties (CHEVALLEY [1]-[3], WEIL [1][1]). Although in most cases we can do without the so-called "loally analytic" theory of CHEVALLEY, it would be sometimes convenient to use this powerful theory to the study of "truely local" properties of algebraic varieties or of differential forms on them. Since we can nowhere find the discussion about the connections between two theories, we shall give it in the following.

At first the word "variety" in the theory of CHEVALLEY is synonymous as the "variety" defined over a fixed algebraically closed field k in the sense of WEIL. If we use the word variety only in this meaning, the notions of a "simple" subvariety in both theories are shown to be the same (cf. CHEVALLEY [3] prop. 4 of Part III, § 3). Furthermore other formal notions in both theories can be seen to be synonymous, and the statement "U and V are subvarieties of a variety Ω and M is a proper component of the intersection $U\cap V$ on Ω" has the same meaning in both theories; thus the symbols $i_{\Omega}(M;\ U\cdot V)$ and $i(U\cdot V,\ M;\ \Omega)$ are at the same time defined.

THEOREM. *It holds* $i_{\Omega}(M;\ U\cdot V)=i(U\cdot V,\ M;\ \Omega)$.

Proof. It can be easily seen that we have only to show $i(P;\ V\cdot L)=j(V\cdot L,\ P)$, where P is a proper point of intersection of a variety V and of a linear variety L in S^n. We shall use n letters $X_1,\dots\dots, X_r, Y_1,\dots\dots, Y_{n-r}$ ($r=\dim(V)$) in describing the equations in S^n. By a suitable affine correspondence, which is everywhere biregular, we may assume that L is defined by the set of equations $X_1=0,\dots\dots, X_r=0$ in S^n, and P has the coordinates $(0,\dots\dots,0)$ (cf. CHEVALLEY [3] th. 7 of Part III, § 5; WEIL [1] th. 10 of Chap. VI, § 3). We denote by

$$m((y)\rightarrow(\bar{y})/(x)\rightarrow(\bar{x})(k))$$

the multiplicity of a specialization $(y)\rightarrow(\bar{y})$ over the specialization $(x)\rightarrow(\bar{x})$ with reference to k, whenever it is defined. Now if $(x,\ y)$ is a generic point

1) Numbers in [ ] refer to the references cited at the end of the paper.