

On the Theory of Algebraic Correspondences and its Application to the Riemann Hypothesis in Function-Fields

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Since the publication of ARTIN's monumental *Quadratische Körper im Gebiete der höheren Kongruenzen*, one of the central problems in the theory of algebraic function-fields has been to prove an analogy of RIEMANN hypothesis for "congruence ζ -functions." Although in "elliptic" case this problem had been solved by HASSE, the general case was beyond the scope of his arithmetical method; and it was André WEIL who first insighted the deep connection between the HURWITZ' formula in the theory of algebraic correspondences and the RIEMANN hypothesis in function-fields. By this discovery a new way to the solution of the RIEMANN hypothesis was opened; he sketched the outline of it in a C. R.-note in 1940, and a year later he published the outline of another proof in a P. N. A. S.-note, depending only on the SEVERI's theory of algebraic correspondences. In this paper we shall develop the algebraic theory of correspondences, centering around the SEVERI's formula on the "virtual degree" of divisors on the product of two algebraic curves, in a most general form based on the WEIL's *Foundations of algebraic geometry* (A. M. S. Coll., v. XXIX, 1946), and as its application, we shall prove the RIEMANN hypothesis following the idea of WEIL's P. N. A. S.-note.¹⁾

During the whole period of my investigation, I was encouraged by Prof. Y. AKIZUKI and by Dr. K. IWASAWA, to whom I express my sincere gratitude, and I wish to dedicate this paper to Prof. S. IYANAGA.

I. Multiplication Ring.

1. *The curve.* We shall fix once for all a "universal domain" K^1 of characteristic p (p is therefore either zero or a positive prime number). Let K be a regular extension of dimension 1 over a perfect field k_0 , or,

1) After my investigation had been completed (in November 1948), I noticed by another P. N. A. S.-note of WEIL (in 1948) that he had also published a detailed proof of his notes in *Pub. Inst. Strassbourg* (N. S., no. 2), pp. 1-85 (1948).

2) We shall use the same terminologies and notations in WEIL's book; the results in the same book will be used without mentioning.